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Hybrid Fast-Ramping Accelerator to 750 GeV/c: Refinement and Parameters over Full Energy Range

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Abstract

Starting with the lattice design specified in [Garren and Berg, MAP-doc-4307, 2011], we refine parameters to get precise dispersion suppression in the straight sections and eliminate beta beating in the arcs. We then compute ramped magnet fields over the entire momentum range of 375 GeV/c to 750 GeV/c, and fit them to a polynomial in the momentum. We compute the time of flight and frequency slip factor over the entire momentum range, and discuss the consequences for longitudinal dynamics.

Our goal is to find the magnet fields as a function of momentum for the lattice described in [1]. We will use these fields to compute the time of flight and frequency slip factor as a function of momentum.

Some precision in our definitions is helpful at this point. All of the magnets will have a field which is a function of time during the acceleration cycle. If we take a snapshot of the magnet fields at a given time and treat them as fixed, then at that time there is a closed orbit which depends on momentum (we are assuming no RF for the purposes of this discussion). Each time has a design momentum, which is monotonically increasing with time, and thus there is a natural mapping from the design momentum to time. We will thus speak of magnet fields and lattice properties as a function of momentum, when we are really speaking of magnet fields at a given time, and lattice properties at a given time at the design momentum for that time, assuming the magnet fields are fixed at their values at that time.

Furthermore, there are properties such as the dispersion and frequency slip factor which are related to derivatives of lattice properties with respect to momentum. These properties depend on time and therefore, via the mapping described int he previous paragraph, on momentum. In defining these quantities, however, we fix the magnet fields at the given values based on the *design* momentum, find the lattice properties as a function of momentum (again, assuming the fields are fixed), and evaluate the derivative with respect to momentum at the design momentum. Thus, in particular, there is not a direct relationship (via differentiation) between the position as a function of the design momentum and the dispersion (as there would be in a cyclotron, for instance).

In this calculation, we maintain the following properties for each design momentum:

- The closed orbit position and transverse momentum is zero in the straight section.
- The dispersion (in position and transverse momentum) is zero in the straight section.
- The phase advance in each arc cell is 90° in both planes.
- The beta functions at corresponding points at the end and center of each arc cell (*i.e.*, halfway between each pair of identical quadrupoles, plus at the corresponding position at the ends of the arcs) are identical. In other words, the lattice functions int he arc are identical whether they are calculated for a single arc cell as a period or for the full superperiod. This will simplify nonlinearity cancellation for the chromatic correction sextupoles.
- Each matching cell, treated as if it were a period, has a phase advance of 90° in both planes.
- The tune of the entire superperiod is 3.25 in both planes.

To accomplish this, we follow a sequence of steps:

1. At every momentum, the arc cell is tuned to have a phase advance of 90° in both planes, and to steer the beam through the centers of the quadrupoles.
2. At the central momentum, in the matching section, the drift length between magnets (other than identical quadrupoles; 0.26322 m in [1]) is adjusted such that with both matching cells identical, starting with the lattice functions at the arc end, the beam is steered through the quadrupole centers, dispersion is zero at the straight section, and the phase advance through the dispersion section is 180°.
3. The quadrupoles and dipoles in the matching section are adjusted so that each matching cell, treated as a periodic cell, has a phase advance of 90°, and the closed orbit and

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Table 1: Cell layout for a half arc cell. Dipoles are rectangular bends. Length parameters are defined in Table 2. Fields as a function of momentum are given in Table 6.

Type	Length	Field
Drift	L0A	
Quadrupole (F)	LQA	B1FA
Drift	L1A	
Dipole (Cold)	LCA	8 T
Drift	L1A	
Dipole (Warm)	LWA	B0A
Dipole (Warm)	LWA	B0A
Drift	L1A	
Dipole (Cold)	LCA	8 T
Drift	L1A	
Dipole (Cold)	LCA	8 T
Drift	L1A	
Dipole (Warm)	LWA	B0A
Dipole (Warm)	LWA	B0A
Drift	L1A	
Dipole (Cold)	LCA	8 T
Drift	L1A	
Quadrupole (D)	LQA	B1DA
Drift	L0A	

Table 2: Length parameters for an arc cell. LCA is more precisely computed as $2\sin(3\pi/1664)p/B$, where B is 8 T, and p is the central momentum of 562.5 GeV/c.

Name	Length (m)
L0A	0.29002
L1A	0.35
LQA	1.60
LWA	3.97319
LCA	≈2.65679

dispersion (both in position and transverse momentum) are zero at the straight section. Each matching cell is individually reflection symmetric.

4. The quadrupoles in the straight section are adjusted to match to the beta functions propagated from the arc through the matching section, and to have a phase advance of 270° for the entire straight. In fact, one should be able to adjust this phase advance over a wide range of values to set the ring tune to a desired value.

The resulting lattice cells are given in Tables 1–5.

We compute the magnet fields at 128 momenta, and perform a least squares fit of the fields to a fifth order polynomial. Table 6 gives the resulting coefficients, and field values at the minimum and maximum momenta. As can be seen, the fields are linear to significantly better than 1%.

Figure 1 shows the aperture required for the beam. The transverse emittance only makes a small contribution to this; it is dominated by the dispersion size and closed orbit variation with momentum. Shorter lattice cells (for a given phase advance per cell) would reduce both the closed orbit variation and the dispersion size. The closed orbit variation could also

Table 3: Cell layout for a half matching cell. Dipoles are rectangular bends. Length parameters are defined in Table 4. Each cell is individually symmetric about the end of this table. Fields as a function of momentum are given in Table 6.

Type	Length	Field
Drift	L0M	
Quadrupole (F)	LQM	B1FM
Drift	L1M	
Dipole (Cold)	LCM	8 T
Drift	L1M	
Dipole (Warm)	LWM	B0FM
Dipole (Warm)	LWM	B0FM
Drift	L1M	
Dipole (Cold)	LCM	8 T
Drift	L1M	
Dipole (Cold)	LCM	8 T
Drift	L1M	
Dipole (Warm)	LWM	B0DM
Dipole (Warm)	LWM	B0DM
Drift	L1M	
Dipole (Cold)	LCM	8 T
Drift	L1M	
Quadrupole (D)	LQM	B1DM
Drift	L0M	

Table 4: Length parameters for a matching cell. LCM is $2\sin(\pi/832)p/B$, similarly to LCA in Table 2.

Name	Length
L0M	0.38665 m
L1M	≈0.27774 m
LQM	4/3 LQA
LWM	2/3 LWA
LCM	≈1.77120 m

Table 5: Cell layout for half the straight. The full straight is reflection symmetric about the end of the table. LS is 28.96996 m. Fields are given in Table 6.

Type	Length	Field
Drift	L0A	
Quadrupole (F)	LQA	QFS1
Drift	LS	
Quadrupole (D)	LQA	QDS1
Drift	2·L0A	
Quadrupole (D)	LQA	QDS1
Drift	LS	
Quadrupole (F)	LQA	QFS2
Drift	2·L0A	
Quadrupole (F)	LQA	QFS2
Drift	LS	
Quadrupole (D)	LQA	QDS2
Drift	L0A	

Table 6: Fields as a function of momentum. Values are given at 375 GeV/c ($p_{\min}$) and 750 GeV/c ($p_{\max}$). Then coefficients of a polynomial in $x = (2p - p_{\min} - p_{\max})/(p_{\max} - p_{\min})$ are given, where b_k is the coeffient of x^k.

Field	At $p_{\min}$	At $p_{\max}$	b_0	b_1	b_2	b_3	b_4	b_5
B1DA (T/m)	-17.4375	-35.1616	-26.3242	-8.8538	0.0217	-0.0072	0.0029	-0.0010
B1FA (T/m)	17.5933	35.2411	26.4219	8.8223	-0.0042	0.0014	-0.0006	0.0002
B0A (T)	-1.78320	1.78316	0.00000	1.78317	-0.00002	0.00001	0.00000	0.00000
B1DM (T/m)	-18.1112	-36.3683	-27.2522	-9.1244	0.0110	-0.0037	0.0014	-0.0005
B1FM (T/m)	18.1896	36.4082	27.3014	9.1085	-0.0022	0.0007	-0.0003	0.0001
B0DM (T)	-1.74504	1.76172	0.00000	1.75621	0.00731	-0.00247	0.00102	-0.00036
B0FM (T)	-1.81358	1.80024	0.00000	1.80464	-0.00585	0.00198	-0.00082	0.00029
B1DS1 (T/m)	-17.6246	-35.2567	-26.4414	-8.8158	0.0006	-0.0002	0.0001	0.0000
B1FS1 (T/m)	18.0305	35.6230	26.8097	8.7969	0.0173	-0.0006	-0.0002	0.0000
B1DS2 (T/m)	-17.6389	-35.2613	-26.4484	-8.8118	-0.0014	0.0005	-0.0002	0.0001
B1FS2 (T/m)	17.4294	35.0759	26.2610	8.8231	-0.0085	0.0002	0.0001	0.0000

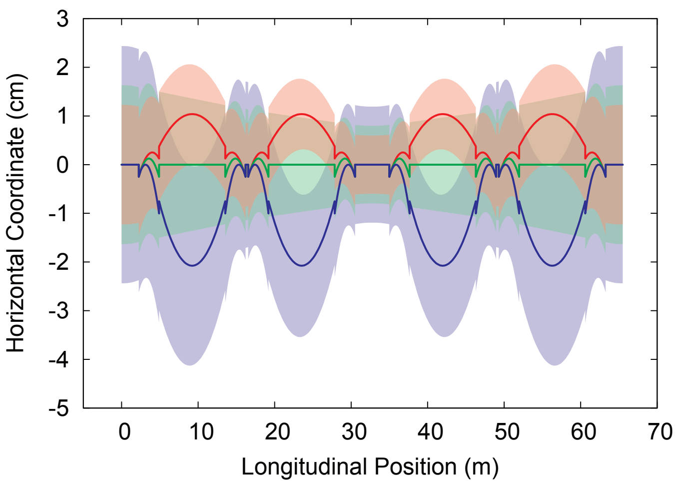

Figure 1: Aperture required for beam in arcs. Blue is for 375 GeV/c, green is for 562.5 GeV/c, and red is for 750 GeV/c. Solid lines are the closed orbits, shaded region is what is required for the beam including the nonzero transverse emittance and energy spread. We assume a 750 MeV energy spread and a normalized transverse emittance of 25 μm. Shaded regions are for 3σ, and are calculated using a linear approximation.

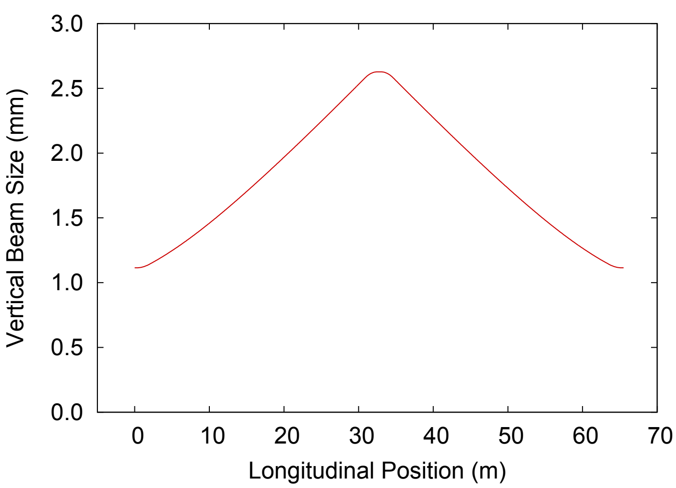

Figure 2: Vertical beam size (3σ) at 375 GeV/c in arcs, calculated using a linear approximation.

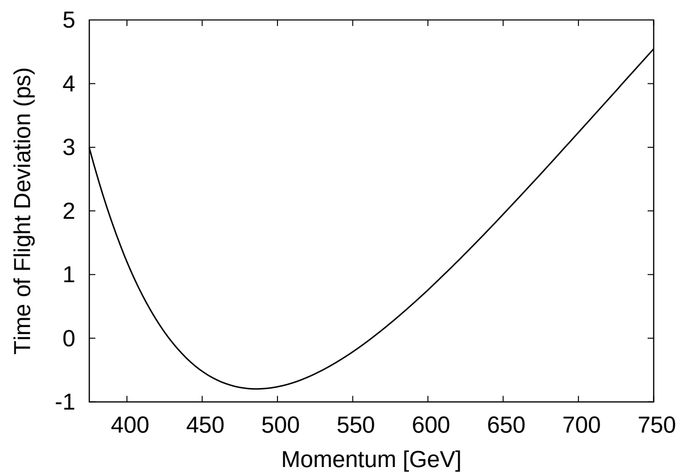

Figure 3: Time of flight per superperiod, relative to the time of flight at the central momentum.

be reduced by using more dipoles per cell. Figure 2 shows the vertical beam size, which is clearly very small.

The time of flight as a function of momentum in this lattice is shown in Fig. 3. The frequency slip factor is 0.00241, and has variation with momentum in the fourth significant digit. In this scenario, the longitudinal dynamics will be governed by the usual longitudinal Hamiltonian for a synchrotron, except the RF phase will vary with time due to the variation of time of flight with momentum (we assume the use of superconducting RF which would not allow a variation of the RF frequency to compensate this variation in time of flight). One could consider reference paritcle motion as for serpentine acceleration in a linear non-scaling FFAG [2]. Only the reference particle would follow the dynamics in [2]; the distribution would be governed by the Hamiltonian for a synchrotron, except that the RF phase varies with time. With sufficient RF voltage (more voltage being needed for higher frequency RF), one could ensure that motion near the reference paritcle is always stable. However, the bucket area would be constantly changing as well, so one would need to maintain a minimum phase (which will also depend on the current particle momentum) to maintain sufficient RF bucket area. The time of flight range is dominated by the variation in path

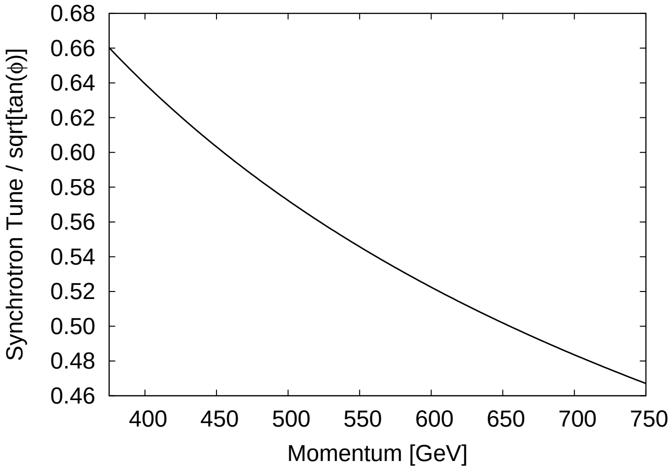

Figure 4: Synchrotron tune for the full ring as a function of momentum. φ is the RF phase off crest.

length, not the variation in particle velocity. The time of flight range would thus be reduced if there was a smaller closed orbit position variation with energy, which would occur if there were shorter cells or more dipoles (alternating between warm and cold) per cell.

For this and subsequent discussions, we assume 1.3 GHz of RF, and acceleration in 24 turns. We would need significantly fewer turns or a much lower RF frequency to use the serpentine reference particle motion described in the previous paragraph. Thus in practice we will need to significantly reduce the time of flight variation with momentum (by reducing the dispersion) or introduce a scheme for correcting the reference paritle time of flight. For subsequent discussions we will ignore this issue.

Assuming a fixed RF phase, we can compute the bucket width as a function of RF phase. The energy width is a minimum at the lowest momentum. The longitudinal beam emittance is 0.25 eV s [3]. If the bucket width is at 4σ, then the required RF phase is 45° off crest. One could run at a phase closer to crest if the momentum compatction were lower, or if there were fewer turns. A lower momentum compaction would require a shorter cells (and therefore more cells in the ring).

Figure 4 gives the synchrotron tune as a function of momentum, which will depend on the RF phase angle. A high synchrotron tune is helpful for stabilizing some collective instabilities. Assuming that we run at the minimum phase angle of 45°, this is not high enough to make the 8 RF stations in the ring necessary; 6 would be sufficient. If we wished to increase the synchrotron tune, we would need to increase the momentum compaction and therefore the dispersion, which is the opposite direction that is desirable for reducing the beam aperture, increasing the RF bucket area, and reducing the time of flight variation with momentum.

Most indications would push us toward a design with shorter cells with a similar phase advance. This would reduce the beam aperture, allow acceleration with a synchronous RF phase closer to crest, and reduce the variation in the time of flight with momentum. The only down side of this is the reduction in the synchrotron tune, which could impact our ability to counteract collective effects. Increasing the number of bending magnets per cell will reduce the required beam aperture and the time of flight variation, and the only real penalty for doing so would be an increase in wasted space for inter-magnet spacing.

For the design as it currently exists, 8 superperiods is not necessary; there is no reason not to go to 6. One would want to keep a similar cell length, since doing otherwise would move the synchronous phase further off crest and increase the beam aperture, the former likely being the most critical problem.

[1] Alper A. Garren and J. Scott Berg, "A Lattice for a Hybrid Fast-Ramping Muon Accelerator to 750 GeV," BNL-96366-2011-IR, MAP-doc-4307 (2011).

[2] J. Scott Berg, "Minimizing longitudinal distortion in a nearly isochronous linear nonscaling fixed-field alternating gradient accelerator," Phys. Rev. ST Accel. Beams **9**, 034001 (2006).

[3] R. B. Palmer, "Draft Muon Collider Parameters Version 1.1," MAP-doc-4318 (2011).